

# **Modeling Vortex-Excited Vibrations of Axially Varying Cylindrical Structures in Non-Uniform Flow Fields**

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## **LONG-TERM GOALS**

Vortex-excited vibrations of bluff cylindrical structures or structural components can, in some cases, lead to reduced fatigue life or even failure of the structure. To examine these possibilities at the design stage, accurate engineering models to predict occurrence and magnitude of vortex-excited vibrations are required. The long-term goal is to develop a robust model for estimating the vortex-excited dynamics of axially varying cylindrical structures in non-uniform flow fields. Recently, we have developed a new method, the inverse-direct method, for estimating these dynamics. The predictions of the inverse-direct method have been compared with experimental data for uniform and tapered pivoted cylinders in uniform and linearly sheared flows. The general agreement between the predictions and the data is quite good. Further comparisons of the predictions of the inverse-direct method with existing experimental data for marine cables and risers in shear flows will allow us to make informed judgments as to the appropriateness of the inverse-direct model for describing vortex-excited vibrations in complex structural and flow situations. A second thrust of the project is the extension of the inverse-direct method to estimating vortex-excited vibrations in an oscillating flow field.

## **OBJECTIVES**

Initially, we explored the extension of wake-oscillator formulations to the modeling of vortex-excited vibrations of axially varying cylindrical structures in non-uniform flow fields. The issues to be addressed were threefold: (1) how best to incorporate axial diffusion of vorticity in wake-oscillator formulations; (2) what effect does the inclusion of axial diffusion of vorticity have on predicted structural responses; and, (3) how well do these predicted responses compare with available experimental data and with what generality. For the most part, we have found that the predicted responses are at considerable variance from experimental results (Balasubramanian *et al.* 2000a, 2000b). Balasubramanian *et al.* have attributed this to the oversimplification of the underlying fluid dynamics that is implicit in nonlinear oscillator models. In the process of examining why the wake-oscillator formulations were unsuccessful, we developed a new method for estimating the vortex-excited vibrations of axially varying cylindrical structures in non-uniform flow fields. This method, which we call the inverse-direct method, successfully predicts the vortex-excited response of uniform and tapered pivoted cylinders in uniform and linearly sheared flows (Skop and Luo 2000). Our objective now is to compare the predictions of the inverse-direct method with experimental data for marine cables and risers.

## APPROACH

An inverse-direct method for predicting the vortex-excited vibrations of uniform and non-uniform cylinders in uniform and non-uniform flows has been developed. In this method, the fluid force acting per unit length on a uniform cylinder in a uniform flow is found by using known experimental results and inverting the equation of motion of the cylinder. This force is a function of the response parameter (structural damping divided by the ratio of displaced fluid mass to structural mass) and the frequency ratio (the ratio of the intrinsic shedding frequency to the structural natural frequency). The dependence of the fluid force on the frequency ratio explains the modal coupling patterns found for taut cables and beams in uniform flows. For non-uniform flows or non-uniform cylinders, the force is applied locally and varies along the cylinder depending on the local values of the response parameter and frequency ratio. The predictions of the inverse-direct method for the vortex-excited vibrations of uniform and tapered pivoted cylinders in uniform and linearly sheared flows have been compared with experimental data. The general agreement between the predictions and the data is quite satisfactory. The next step is to compare the predictions of the inverse-direct method with experimental data for marine cables and risers. Overall, the inverse-direct method is easier to apply and, thusfar, yields more accurate predictions than the previously used nonlinear oscillator models.

## WORK COMPLETED

We have completed the development of the inverse-direct method for estimating the vortex-excited dynamics of axially varying cylindrical structures in non-uniform flow fields. In the inverse portion of the method, the equation of motion of a uniform cylinder in a uniform flow is inverted and known experimental results are used to determine the fluid force per unit length acting on the cylinder. We then find that the vortex-induced oscillating lift coefficient  $C_L$  is determined as (Skop and Luo 2000)

$$C_L = \begin{cases} 0 & \text{if } V_{r1} > V_r > V_{r2} \\ \frac{2(S_G + \alpha\Omega_S^3)A_{\max}(S_G)H(V_r)}{\Omega_S^2} & \text{if } V_{r1} \leq V_r \leq V_{r2} \end{cases} \quad (1)$$

In equation (1),  $S_G$  is the response parameter for the system and  $\alpha$  is the stall parameter. The frequency ratio  $\Omega_S$  is given by  $\Omega_S = \omega_s/\omega_n$  where  $\omega_s$  is the intrinsic Strouhal frequency and  $\omega_n$  is the structural natural frequency. The reduced velocity  $V_r$  is found from

$$V_r = \frac{2\pi V}{\omega_n D}, \quad (2)$$

where  $V$  is the flow velocity and  $D$  is the diameter of the cylinder. Also,  $V_{r1}$  and  $V_{r2}$  are the reduced velocities at the beginning and end of the lock-in region, respectively. The universal response function  $A_{\max}(S_G)$  is determined from

$$A_{\max} = \exp(-0.938S_G). \quad (3)$$

The universal shape function  $H(V_r)$  is prescribed as

$$H = (V_r - V_{r1})(V_r - V_{r2})(\gamma V_r + \lambda), \quad (4)$$

where the coefficients  $\gamma$  and  $\lambda$  are defined by

$$\gamma = \frac{V_{r1} + V_{r2} - 2V_{rp}}{(V_{r1} - V_{rp})^2 (V_{r2} - V_{rp})^2}, \quad (5)$$

and

$$\lambda = \frac{V_{r1} V_{r2} - 2(V_{r1} + V_{r2})V_{rp} + 3V_{rp}^2}{(V_{r1} - V_{rp})^2 (V_{r2} - V_{rp})^2}. \quad (6)$$

In these definitions,  $V_{rp}$  is the reduced velocity at which  $H(V_r) = 1$ . For air, we use  $V_{r1} = 5$ ,  $V_{r2} = 8$  and  $V_{rp} = 6$ . For water, we select  $V_{r1} = 5$ ,  $V_{r2} = 9$  and  $V_{rp} = 6.5$ .

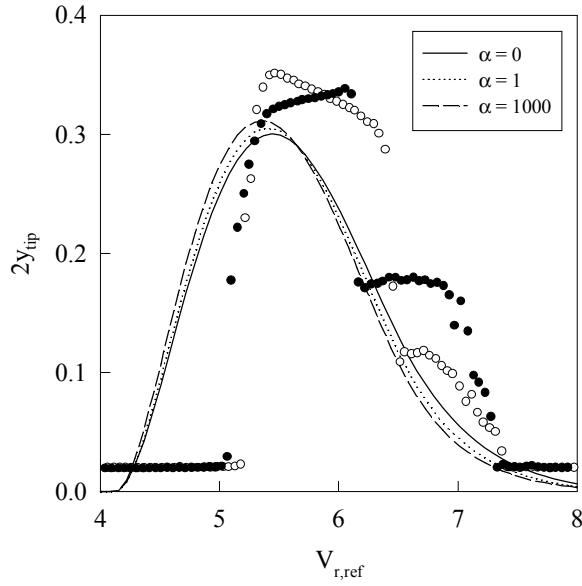
In the direct portion of the inverse-direct method, equation (1) is applied locally along the cylinder. For non-uniform flows or non-uniform cylinders, the force varies along the cylinder depending on the local values of the response parameter and the frequency ratio.

## RESULTS

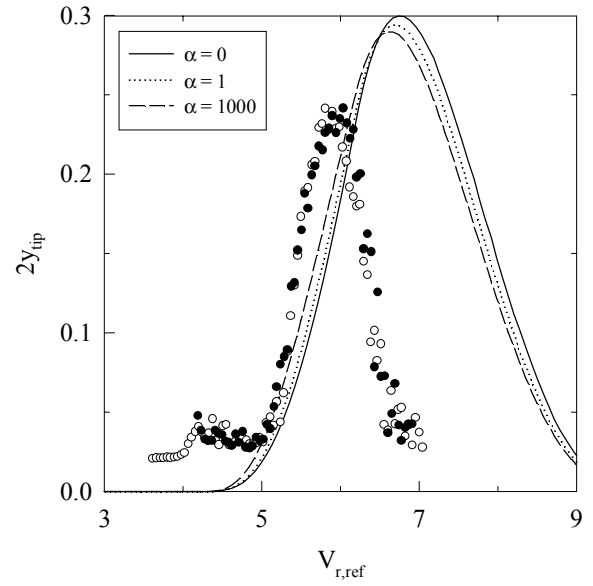
To obtain data against which to test the inverse-direct method, collaborative experiments with the University of Notre Dame have been carried out. The experiments examined the vortex-induced vibrations of uniform and non-uniform pivoted cylinders in uniform and shear flows. The experiments are detailed in Balasubramanian *et al.* (2000a, 2000b).

Predictions of the inverse-direct method and experimental data are shown in Fig. 1 for uniform cylinders in sheared flows and in Fig. 2 for tapered cylinders in uniform flows. In these figures,  $y_{tip}$  is the displacement at the tip of cylinder, made dimensionless by the reference diameter  $D_{ref}$ . Also  $V_{r,ref}$  is the reference reduced velocity defined by  $V_{r,ref} = V_{ref} / (f_n D_{ref})$ . Here,  $V_{ref}$  is the reference velocity measured in the wind tunnel and  $f_n$  is the natural frequency of the pivoted cylinder. For the uniform cylinder (Fig. 1),  $D_{ref} = 0.05715$  m while for the tapered cylinder (Fig. 2) it is  $D_{ref} = 0.0381$  m. The parameter  $\alpha$  is the stall coefficient and the best results have been found with  $\alpha = 1$ .

When all of the cases are considered, the agreement between the predictions and the data is quite good, especially when account is taken of the scatter in the data for uniform cylinders in uniform flows. The predicted peak vibration amplitudes range from  $-11\%$  to  $+25\%$  of the measured ones. The predicted flow velocities at which the peak amplitudes occur are within  $-6\%$  to  $+16\%$  of the measured flow velocities. The main discrepancy between the predictions and the measured data is in the extent of the lock-in regions; the predicted extents tending to be somewhat wider than the measured ones. It should be noted, however, that for uniform flow over a uniform cylinder the lock-in region measured by Balasubramanian *et al.* (2000b) was somewhat narrower than is usually observed.

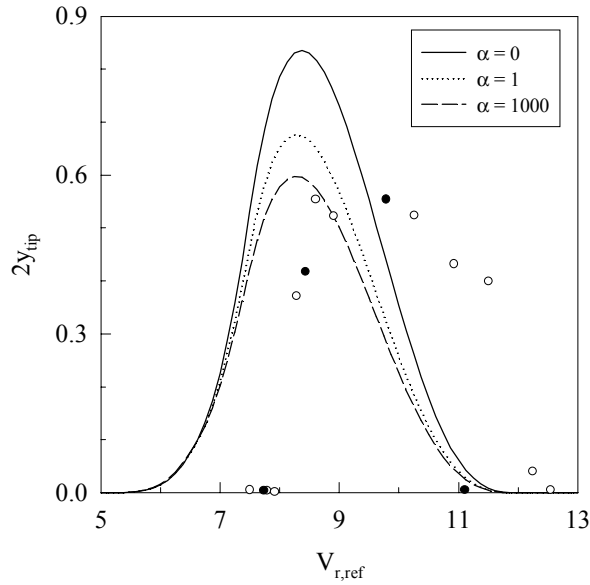


(a)

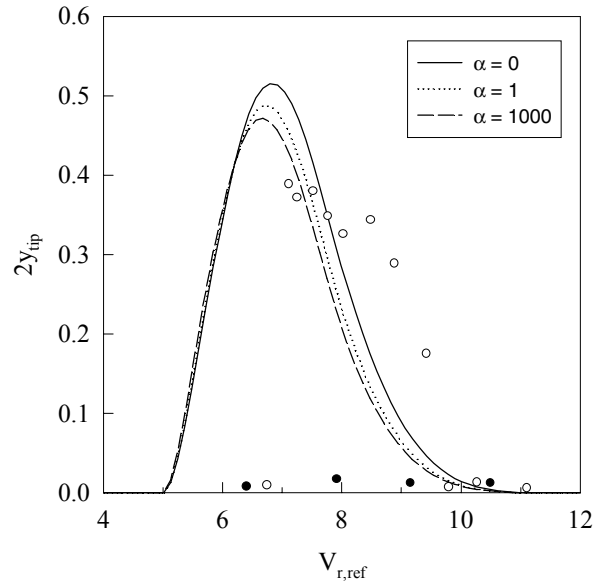


(b)

**Figure 1. Predicted and measured responses for a uniform pivoted cylinder in linearly sheared flows.**  $\circ$ , measurements taken with  $V_{r,ref}$  increasing;  $\bullet$ , measurements taken with  $V_{r,ref}$  decreasing; full curves, predicted responses for various values of  $\alpha$ . (a) Minimum flow velocity at pivot; (b) maximum flow velocity at pivot.



(a)



(b)

**Figure 2. Predicted and measured responses for a tapered pivoted cylinder in a uniform flow.**  $\circ$ , measurements taken with  $V_{r,ref}$  increasing;  $\bullet$ , measurements taken with  $V_{r,ref}$  decreasing; full curves, predicted responses for various values of  $\alpha$ . (a) Pivot at small diameter end; (b) pivot at large diameter end.

## IMPACT/APPLICATIONS

Overall, the inverse-direct is easier to apply and yields more accurate predictions than the previously used nonlinear oscillator models. Because of these properties, the inverse-direct method will quickly replace the nonlinear oscillator models for estimating the vortex-excited dynamics of axially varying cylindrical structures in non-uniform flow fields. However, further validation is required. The next step is to compare the predictions of the inverse-direct method with experimental data for marine cables and risers. We plan on addressing this problem in the coming year.

## TRANSITIONS

No transitions have been accomplished to date. However, once the inverse-direct method is published and further validated, we anticipate that it will be incorporated in Navy and offshore industry design codes much as the original wake oscillator model was some twenty years ago.

## RELATED PROJECTS

We are collaborating closely with the University of Notre Dame on their project “Experiments on vortex-excited oscillations of axially varying cylinders in shear flow,” (Principal Investigator: Albin A. Szewczyk; Co-Principal Investigator: Richard A. Skop). We are also collaborating with Pratap Vanka of the University of Illinois, and his student Gang Luo, on the development of the inverse-direct method.

## REFERENCES

S. Balasubramanian, F.L. Haan, Jr., A.A. Szewczyk and R.A. Skop, 2000a. An experimental investigation of the vortex-excited vibrations of axially varying pivoted cylinders in uniform and shear flow, *Journal of Wind Engineering and Industrial Aerodynamics*, **in press**.

S. Balasubramanian, R.A. Skop, F.L. Haan, Jr., and A.A. Szewczyk, 2000b. Vortex-excited vibrations of uniform pivoted cylinders in uniform and shear flow, *Journal of Fluids and Structures*, **14**, 65-85.

R.A. Skop and G. Luo, 2000. An inverse-direct method for predicting the vortex-excited vibrations of cylinders in uniform and non-uniform flows, *Journal of Fluids and Structures*, **in review**.

## PUBLICATIONS

Balasubramanian, S., 1998. Vortex-excited vibrations of pivoted cylinders in uniform and shear flows, *Ph.D. Dissertation*, University of Miami.

Balasubramanian, S., F.L. Haan, Jr., A.A. Szewczyk and R.A. Skop, 1998. On the existence of a critical shear parameter for cellular vortex shedding from cylinders in nonuniform flow, *Journal of Fluids and Structures*, **12**, 3-15.

Balasubramanian, S., F.L. Haan, Jr., A.A. Szewczyk and R.A. Skop, 2000. An experimental investigation of the vortex-excited vibrations of axially varying pivoted cylinders in uniform and shear flow, *Journal of Wind Engineering and Industrial Aerodynamics*, **in press**.

Balasubramanian, S. and R.A. Skop, 1997. Modeling vortex-excited vibrations of uniform cylinders in uniform and shear flow, *Proceedings of ASME 4th International Symposium on Fluid-Structure Interactions, Aeroelasticity, Flow-Induced Vibration & Noise*, Dallas, TX.

Balasubramanian, S. and R.A. Skop, 1999. Vortex-excited dynamics of a tapered pivoted cylinder in uniform and shear flows, *Proc. 13th ASCE Engineering Mechanics Division Conference*, Baltimore, MD.

Balasubramanian, S., R.A. Skop, F.L. Haan, Jr., and A.A. Szewczyk, 2000. Vortex-excited vibrations of uniform pivoted cylinders in uniform and shear flow, *Journal of Fluids and Structures*, **14**, 65-85.

R.A. Skop and S. Balasubramanian, 1995. A nonlinear oscillator model for vortex shedding from a forced cylinder. Part 1: Uniform flow and model parameters, *International Journal of Offshore and Polar Engineering*, **5**, 251-255.

Skop, R.A. and S. Balasubramanian, 1995. A nonlinear oscillator model for vortex shedding from a forced cylinder. Part 2: Shear flow and axial diffusion, *International Journal of Offshore and Polar Engineering*, **5**, 256-260.

Skop, R.A. and S. Balasubramanian, 1997. A new twist on an old model for vortex-excited vibrations, *Journal of Fluids and Structures*, **11**, 395-412.

Skop, R. A. and G. Luo, 2000. An inverse-direct method for predicting the vortex-excited vibrations of cylinders in uniform and non-uniform flows, *Journal of Fluids and Structures*, **in review**.

Szewczyk, A.A., S. Balasubramanian, F.L. Haan and R.A. Skop, 1997. Experiments on vortex-induced vibrations of pivoted circular cylinders, *Proceedings of ASME 4th International Symposium on Fluid-Structure Interactions, Aeroelasticity, Flow-Induced Vibration & Noise*, Dallas, TX.